A Sequential Signalling Model of Convertible Debt Call Policy

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Outline

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- For bondholders: they are willing to convert if they believe the stock price will soar in the future.
- For issuer: they are willing to call the bond if they believe they can refinance with lower interest.

Puzzles in Convertible bond calls

- Although it has been shown that conversion of these bonds should optimally be forced as soon as this is feasible, actual calls are significantly delayed relative to this prescription.
- Common stock returns are significantly negative around the announcement of the call of a convertible debt issue. But why managers take actions which result in a decline in the value of the common stock?
- In this paper, we attempt to explain delayed calls and negative stock returns at call announcement using a sequential information signalling approach.

Basic idea of the game

- Managers periodically receive private information regarding the firm's prospects.
- Manager's compensation is assumed to depend on the stock price in both current and future periods.
- Each period, the manager must decide whether or not to force the conversion.
- The market tries to infer the private information of managers by observing their call decisions.
- Manger also considers inference of the market.

Signalling game model

- Assumption: (1) No dividend risk-free zero-coupon bonds. (2) Firm's securities prices depend only on the extent to which the manager's private information is signalled to the market by his or her call policy. (3) Conversion can be forced at any time prior to maturity of the bond. (4) No default. (5) call price at $t = 1, 2$ is zero.
- There are 4 dates indexed by $t = 0.1, 2, 3$.
- The total dollar payoff of the firm at $t = 3$ is either $\hat{\delta} = H$ or $\hat{\delta} = L$. This is common knowledge.
- At dates 1 and 2, the manager receives private information regarding the payoffs at $t = 3$. The message which arrives at t is denoted as $\hat{\phi}_t$, which can take value G(Good) or B(Bad).
- The joint distribution $P(\phi_1 \phi_2 \delta)$ is common knowledge.

Signalling game model

- At $t = 0$, the firm has outstanding N shares of stock and M callable, convertible bonds. Each bond promises to pay \$1 at $t = 3$ unless it havs been converted to stock.
- Each bond can be converted at any date into r(conversion rate) shares of stock.
- The stockholders of the firm receive the payoff to the firm net of any payments to bondholders.
- To make sure that in some cases the bond may be converted, we need have constraint: $r\frac{L}{N+Mr} < 1 < r\frac{H}{N+Mr}$, denoted as $rl < 1 < rh$.
- Manager's compensation (utility) function is: $W = \omega(S_1) + \omega(S_2) + \omega(S_3)$, where S_t is the stock price at time t and ω is any concave and strictly increasing function.

Signalling game model

- Clearly, there is no reason to convert voluntarily before date 3, because there is no reward for doing that. We can wait for more information.
- $\rightarrow$ we can wait until $t = 3$, if it is H, we convert, otherwise we don't convert.
- $\rightarrow$ bondholders will always convert if the debt is called, otherwise we receive nothing (call price=0). That's called forced conversion.

The conversion game

- Players are firm's manager and investors. Manager's actions is $d_t = \{C(call), N(notcall)\}$, provided that they have not been called or converted previously. Define $l_N = (L - M)/N$
- Denote the payoffs per share by $s(d_1 d_2 \hat{\delta})$ and the payoffs to each convertible by $k(d_1 d_2 \hat{\delta})$.

Payoffs Per Share (s) and Per Bond (k) as Functions of Manager's Call Decisions ($d_1 d_2$) and the Firm's Payoff ($\tilde{\delta}$)

Payoffs to Each Share (s)			Payoffs to Each Convertible (k)		
$\tilde{\delta}$ \ $d_1 d_2$	C or NC	NN	$\tilde{\delta}$ \ $d_1 d_2$	C or NC	NN
H	h	h	H	rh	rh
L	l	l_N	L	rl	l

The conversion game - Mixed strategy

- Manager plays mixed strategies at $t = 1, 2$ with probability measure

$$\sigma_1(d_1|\phi_1) \tag{1}$$

is the probability of choosing d_1 in $\{C, N\}$ given message ϕ_1.

$$\sigma_2(d_2|\phi_1\phi_2 d_1) \tag{2}$$

is the probability of choosing d_2 in $\{C, N, noaction\}$ given messages ϕ_1, ϕ_2 and previous action d_1.

The conversion game - stock price

- stock price reflects the market belief, the market values of the stock S_t at $t = 0, 1, 2$ are given by the expected values of their payoffs given current beliefs and the strategies:

$$S_2(d_1 d_2) = \sum_\delta P(\delta \mid d_1 d_2) s(d_1 d_2 \delta), \tag{3}$$

$$S_1(d_1) = \sum_{\phi_1 \phi_2 d_2} S_2(d_1 d_2) \sigma_2(d_2 \mid \phi_1 \phi_2 d_1) P(\phi_1 \phi_2 \mid d_1), \tag{4}$$

$$S_0 = \sum_{\phi_1 d_1} S_1(d_1) \sigma_1(d_1 \mid \phi_1) P(\phi_1). \tag{5}$$

The conversion game - definition

- Definition: An equilibrium of the conversion game described above is (a) a set of strategies for the manager, $\sigma_1(d_1|\phi_1)$ and $\sigma_2(d_2|\phi_1\phi_2 d_2)$, which define the probabilities of forced conversion given previous messages and decisions, and (b) a set of investor's beliefs, P, which give the posterior probabilities assigned by investors to messages and outcomes, given observed managerial decisions such that
- I. Beliefs of investors are consistent with Bayes' rule given strategies, σ_1 and σ_2, and past decisions, d_1, and d_2, i.e.,

$$P(\phi_1\phi_2\delta|d_1) = \frac{\sigma_1(d_1|\phi_1)P(\phi_1\phi_2\delta)}{\sum_{x=G,B}\sigma_1(d_1|x)P(x)} \tag{6}$$

$$P(\phi_1\phi_2\delta|d_1d_2) = \frac{\sigma_1(d_2|\phi_1\phi_2 d_1)P(\phi_1\phi_2\delta)}{\sum_{x,y=G,B}\sigma_2(d_2|xyd_1)P(xy|d_1)} \tag{7}$$

The conversion game - definition

II. The manager's strategy maximizes his or her welfare at each stage given market values of the shares at each date as functions of investors' beliefs as determined in (I) and given future strategies of the manager, i.e., at date 2, for any $\phi_1\phi_2$, $x = \sigma_2(C \mid \phi_1\phi_2 N)$ solves

$$m(\phi_1\phi_2) \equiv \max_{0\le x\le 1} x[w(S_2(NC) + E(w(s) \mid \phi_1\phi_2 NC)] + (1-x)[w(S_2(NN)) + E(w(s) \mid \phi_1\phi_2 NN)], \quad (8)$$

where S_1 is given by (4) with P given by (7),

$$\sigma_2(N \mid \phi_1\phi_2 N) = 1 - \sigma_2(C \mid \phi_1\phi_2 N), \quad (9)$$

$$\sigma_2(N \mid \phi_1\phi_2 C) = \sigma_2(C \mid \phi_1\phi_2 C) = 0,\ \sigma_2(\text{no action} \mid \phi_1\phi_2 C) = 1 \quad (10)$$

and at date 1, for any ϕ_1, $x = \sigma_1(C \mid \phi_1)$ solves

$$\max_{0\le x\le 1} x[2w(S_1(C) + E(w(s) \mid \phi_1 C)] + (1-x)[w(S_1(N)) + E(m(\phi_1\phi_2) \mid \phi_1)], \quad (11)$$

$$\sigma_1(N \mid \phi_1) = 1 - \sigma_1(C \mid \phi_1). \quad (12)$$

The conversion game - definition comparison

- Requirements of SE:
  1. (Sequential Rationality) The response Π (probability distribution over actions r) to out-of-equilibrium move m is a best response given beliefs

$$\Pi(r, m) \in BR(\mu(\tau|m), m)$$

  2. (Consistency) defecting with out-of-equilibrium move m is not incentive compatible given the response Π:

$$\sum_r u(\tau, m, r)\Pi(r, m) \leq u^*(\tau) \, \forall \tau$$

  where $u^*(\tau)$ is the equilibrium utility of type τ.
  3. Beliefs for on-equilibrium messages are calculated using equilibrium strategies and Bayes rule

The conversion game - Nonsignalling Equilibria

- If no information is conveyed by the call policy. (e.g. $\sigma_1(C|\phi_1)$ is constant for all ϕ_1)

THEOREM 1 (NONSIGNALLING EQUILIBRIUM). *An equilibrium of the conversion game is given by:*

I. Investors do not infer any information from the call decisions of the firm, i.e.,

$$P(\phi_1\phi_2\delta \mid d_1) = P(\phi_1\phi_2\delta) \quad \text{for all} \quad \phi_1\phi_2\delta \quad \text{and} \quad d_1,$$

$$P(\phi_1\phi_2\delta \mid d_1d_2) = P(\phi_1\phi_2\delta) \quad \text{for all} \quad \phi_1\phi_2\delta \quad \text{and} \quad d_1d_2.$$

II. The manager always calls at date 1 independent of his or her private information, i.e.,

$$\sigma_1(C \mid \phi_1) = 1,$$

$$\sigma_2(\text{no action} \mid \phi_1\phi_2 C) = 1,$$

$$\sigma_2(C \mid \phi_1\phi_2 N) = 1.^{16}$$

- $\rightarrow$ The convertibles will be called in order to force conversion as soon as this is feasible.
- But this equlibria doesn't expain the puzzles.

The conversion game - Signalling Equilibria

- Investors believe that managers call only when their private message is negative. (e.g. $\sigma_1(C|\phi_1) = 1$ if $\phi_1 = B$, $\sigma_1(C|\phi_1) = 0$ if $\phi_1 = G$)

THEOREM 2 (SIGNALLING EQUILIBRIUM). *There exist values of the exogenous parameters, $w(\cdot)$, H, L, N, M, r, and the prior beliefs such that the following posterior beliefs and strategies constitute an equilibrium of the conversion game:*

I. Investors believe that if the manager calls (does not call) at a given date, he or she has received a bad (good) message at that date, i.e.,

$$P(G \mid N) = 1, \qquad P(B \mid C) = 1, \qquad P(GG \mid NN) = 1,$$
$$P(GB \mid NC) = 1, \qquad P(B\phi_2 \mid C) = P(\phi_2 \mid B), \qquad \text{for each} \quad \phi_2.$$

II. The manager always calls after receiving a "bad" message and never calls after receiving a "good" message, i.e.,

$$\sigma_1(N \mid G) = 1, \qquad \sigma_1(C \mid B) = 1,$$
$$\sigma_2(N \mid GGN) = \sigma_2(N \mid BGN) = 1,$$
$$\sigma_2(C \mid GBN) = \sigma_2(C \mid BBN) = 1,$$
$$\sigma_2(\text{no action} \mid \phi_1\phi_2 C) = 1.$$

Finally, there exist parameters such that prior probabilities are not degenerate, i.e., such that

$$0 < P(H \mid GG) < 1, \qquad 0 < P(G \mid G) < 1, \qquad 0 < P(G) < 1,$$

- *and the above beliefs and strategies still constitute an equilibrium.*

The conversion game - Signalling Equilibria

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- Why manager do nothing when receiving Good news?
  - It conveys a signal to investors that manager's message is positive, resulting in a higher market value at $t = 1, 2$. Thess are benefits for the manager.
  - Costs: if manager doesn't call before $t = 3$, and $\delta = L$, he will get loss ($l_N < l$).
  -But Good news give manager confidence $\delta = L$ almost never happen. So manager doesn't call. (Puzzle 1)

The conversion game - Signalling Equilibria

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L	l	l_N	L	rl	1

- Why manager call when receiving Bad news?
  - It conveys a signal to investors that manager's message is negative, resulting in a lower market value at $t = 1, 2$. (Puzzle 2) These are costs for the manager.
  - Benefits: if manager call before $t = 3$, the stock price will be much better at $t = 3$.
  - The negative message lead higher probability of $\sigma = L$. So manager does call to ensure the welfare at $t = 3$.

Conclusions

- The effects of calling on the current stock price are
  - (a) a decrease due to the unfavorable signal conveyed and
  - (b) an increase due to the increase in share payoffs at date 3 due to forced conversion.
- The manager balance the cost and benefit based on the information she received, which leads the two puzzles in convertible bonds market.

Thanks!

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