Kyle(1985) Notes

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1 Overview

This note is to summarize the core idea of Kyle (1985) in "Continuous Auctions and Insider Trading".

2 Model Setup

- The unconditional distribution of asset follows

$$V \sim N(\mu, \sigma_V^2) \tag{1}$$

- This is a sequential game, where traders move first, then market maker set the price and fill the orders.
- There are two types of traders.

  Noise trader has random demand,

$$u \sim N(0, \sigma_u^2). \tag{2}$$

  Speculator has private observation about V, speculator chooses her optimal demand x to maximize her expected payoff.

- u and V are independent.
- Market maker only observes order batches $q = x + u$. Market maker is risk neutral and competitive, which leads zero profit, in other words, the price will be set as

$$p = E[V|q]. \tag{3}$$

3 Solution

- Let's look for equilibrium where speculator's strategy is linar:

$$x = \beta(V - \mu), \tag{4}$$

where $\beta > 0$ measures speculator aggression.

- Market maker knows the speculator's strategy and observes

$$q = x + u = \beta(V - \mu) + u. \tag{5}$$

Recalling that u and V are independent. The conditional and unconditional distribution of q then follows,

$$q|V \sim N(\beta(V - \mu), \sigma_u^2) \tag{6}$$

$$q \sim N(0, \beta^2\sigma_V^2 + \sigma_u^2) \tag{7}$$

- Market maker updates her belif about the V by Bayesian rule[1],

$$\begin{aligned} f(V|q) &= \frac{f(V)f(q|V)}{f(q)} \\ &= \frac{k_1 k_2}{k_3} exp\{-[\frac{(q - \beta(V - \mu))^2}{2\sigma_u^2} + \frac{(V - \mu)^2}{2\sigma_V^2}] + \frac{q^2}{2(\beta^2\sigma_V^2 + \sigma_u^2)}\} \\ &= \frac{k_1 k_2}{k_3} exp\{-\frac{\frac{1}{\sigma_V^2} + \frac{\beta^2}{\sigma_u^2}}{2}(V - \mu - \frac{\beta\sigma_V^2}{\beta^2\sigma_V^2 + \sigma_u^2}q)^2 - k_4 + \frac{q^2}{2(\beta^2\sigma_V^2 + \sigma_u^2)}\} \\ &= k \cdot exp\{-\frac{\frac{1}{\sigma_V^2} + \frac{\beta^2}{\sigma_u^2}}{2}(V - \mu - \frac{\beta\sigma_V^2}{\beta^2\sigma_V^2 + \sigma_u^2}q)^2\} \end{aligned} \tag{8}$$

$$V|q \sim N(\mu + \frac{\beta\sigma_V^2}{\beta^2\sigma_V^2 + \sigma_u^2}q, \frac{1}{\frac{1}{\sigma_V^2} + \frac{\beta^2}{\sigma_u^2}}) \tag{9}$$

- Equation (3) can be rewritten as,

$$p = E[V|q] = \mu + \frac{\beta\sigma_V^2}{\beta^2\sigma_V^2 + \sigma_u^2}q := \mu + \lambda q \tag{10}$$

where $1/\lambda$ is a measure of market depth.

[1] Where $k_1, k_2, k_3, k_4, \& k$ does not depend on V

- To verify the Bayesian Nash equilibrium of $\{x = \beta(V-\mu), p = \mu+\lambda q\}$, We need to solve the β and λ, s.t strategy in equation (4) is the best response given equation (10).
- The expected payoff of speculator is,

$$E[(V-p)x|V] = E[(V-\mu-\lambda(x+u))x|V] = x(V-\mu-\lambda x) \quad (11)$$

$$F.O.C \Rightarrow x^* = \frac{V-\mu}{2\lambda} \quad (12)$$

$$(4)\&(12) \Rightarrow \beta = \frac{1}{2\lambda} \quad (13)$$

$$(10)\&(13) \Rightarrow \frac{1}{2\beta} = \frac{\beta\sigma_V^2}{\beta^2\sigma_V^2 + \sigma_u^2}, \quad (14)$$

which yields

$$\beta = \frac{\sigma_u}{\sigma_V} \quad (15)$$

$$\lambda = \frac{\sigma_V}{2\sigma_u} \quad (16)$$

- Th equilibrium is $\{x^* = \frac{\sigma_u}{\sigma_V}(V-\mu), p^* = \mu + \frac{\sigma_V}{2\sigma_u}q\}$

4 Discussion

- The aggression of specultor is measured by $\beta = \frac{\sigma_u}{\sigma_V}$ can be explained as:

  (1) more noise traders $\sigma_u \uparrow \rightarrow$ more aggressive,

  (2) more asymmetric information $\sigma_V \uparrow \rightarrow$ less aggressive.
- The ex ante unconditional expected payoff of speculator is

$$E[x(V-\mu-\lambda x)] = \beta\frac{\sigma_V^2}{2} = \frac{\sigma_V\sigma_u}{2} \quad (17)$$

where the uncertainty of V, with the noise of u can hide speculator's information and increases the expected payoff.
- Market maker's ex post conditional variance after observing q is,

$$V[V|q] = \frac{1}{\frac{1}{\sigma_V^2} + \frac{\beta^2}{\sigma_u^2}} = \frac{\sigma_V^2}{2} \quad (18)$$

Excatly half the pervious variance: Insider reveals half his information.

- Compare to Glosten-Milgrom(1985): in Kyle orders are cleared by the market maker in batches rather than one-by-one. So the bid ask spread is not implemented under this framework. Some adjustment can applied to include the bid ask spread framework in discrete Kyle's variant model[2].

5 Reference

- Kyle, A.S., 1985. Continuous auctions and insider trading. Econometrica: Journal of the Econometric Society, pp.1315-1335.
- Lecture 5 `https://starkov.site/teaching.html`
- `https://sites.duke.edu/xjiang/files/2023/05/Class_2.pdf`
- `https://home.cerge-ei.cz/petrz/fm/f400n30.pdf`

[2]Reference here: `https://sites.duke.edu/xjiang/files/2023/05/Class_2.pdf`